Teacher notes
Topic C

Another missing equation.

The new Guide does not explicitly state the relation between intensity and amplitude of a wave. Yet this is crucial in understanding interference. The term “intensity” is used four times in the Guide in Topic C and twice in Topic B but it is never defined. The Guide mentions the term “apparent brightness” in Topic B which is what astronomers call intensity but the crucial connection between intensity and amplitude is missing.

In Topic C, HL, we learn that the total energy of a body performing SHM is given by $E_T = \frac{1}{2}m\omega^2 x_0^2$. This means that the energy of the oscillating system is proportional to the amplitude squared. It is then reasonable to expect that the energy and hence the intensity, carried by a travelling wave is also proportional to the amplitude squared. Calling the amplitude, *A*, we have $I = kA^2$ where *k* is a constant that depends on the type of wave we are discussing. We need this relation to be able to understand questions such as:

1. Two sources emit coherent waves in phase. At point P we observe constructive interference. The intensity at P due to one source alone is *I*. What is the intensity at P?

Since we have constructive interference at P the amplitude at P is 2*A* where *A* is the amplitude at P due to one source alone. Hence $I = kA^2$. The total intensity is then $I' = k(2A)^2 = 4kA^2 = 4I$; the intensity increases by a factor of 4.

In other words, if the sources are sources of sound the loudness at P is 4 times the loudness at P from one source alone. This is an interesting and curious fact: naively, we would expect that the intensity would be twice the intensity from just one source alone, but this is not correct. We would get twice the intensity only if the sources were incoherent. If they are coherent, we get 4 times the intensity. So, the relation $I = kA^2$ is crucial in any discussion of coherence and why coherence is needed to observe interference. But this brings up questions of energy conservation: where does the extra energy come from? At points of destructive interference, the amplitude is zero, so the intensity is zero. This resolves the energy conservation problem: The original available intensity at the sources is 2*I* where *I* is the intensity of one source. At points of destructive interference, the intensity is 0 and at points of constructive interference the intensity is 4*I*. The average of 0 and 4*I* is 2*I* and energy is conserved. Interference changes the distribution of energy.

2. Two sources emit coherent waves in phase. At point P we observe constructive interference. The intensity at P is *I*. One source is removed. What is the intensity at P now?

This is just the previous problem in reverse. Since we have constructive interference at P the amplitude at P is 2A where A is the amplitude at P due to one source alone. Hence $I = k(2A)^2 = 4kA^2$. When one source is removed the amplitude at P is now just A and so the intensity there is $I' = kA^2 = \frac{1}{4}4kA^2 = \frac{I}{4}$.

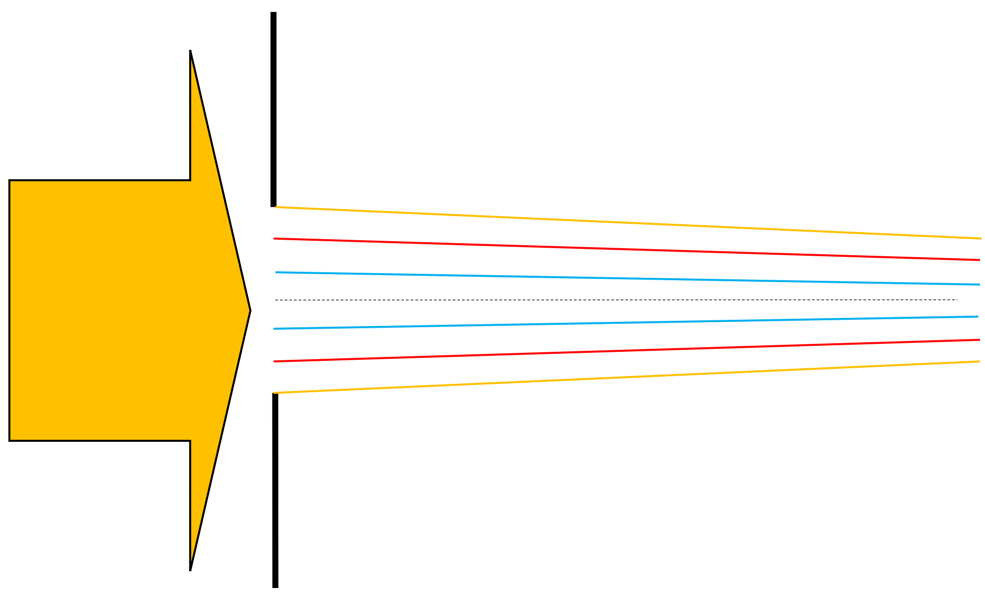
3. Two identical flashlights are pointed at the same point P on a wall and are equidistant from P. The intensity at P from one flashlight alone is 20 W m^{-2}. What is the intensity at P when both flashlights point at P?

The two flashlights are incoherent, so no interference is observed. The intensity will be 40 W m^{-2}.

4. HL

Coherent monochromatic light is incident on a single slit. The intensity of light on a screen at a point M directly across the midpoint of the slit is 60 W m^{-2}. The slit width is halved. What is the intensity at M now?

At M we have constructive interference between pair of rays in the same color as shown in the figure:



If the amplitude at M from one ray alone is A then the total amplitude is $N \times 2A$ where N is the total number of rays through the slit. Hence $60 = k(2NA)^2 = 4kN^2A^2$. With half the slit width, N is halved and so the new intensity is

$$I = k(2\frac{N}{2}A)^2 = N^2kA^2 = \frac{1}{4} \times 4N^2kA^2 = \frac{60}{4} = 15 \text{ W m}^{-2}$$